The impact of central bank digital currency on bank deposits and the interbank market*

Benjamin Hemingway[†]

November 2024

Abstract

This paper proposes a theoretical model in which a central bank digital currency (CBDC) and bank deposits are imperfect substitutes. Deposits are subject to liquidity shocks. In the absence of a CBDC, the interbank market can redistribute liquidity between banks. The introduction of CBDC leads to a greater dependence of the banking sector on central bank standing facilities, raising the costs of bank funding. Calibrating the model to the Eurozone, the model shows that adjusting the remuneration rate of CBDC has little pass-through to the deposit rate set by banks and also has implications for the transmission of monetary policy especially if the CBDC is unremunerated

Keywords: central bank digital currency, banking, money, interbank market

JEL codes: E42, E52, E58, G21

*I would like to thank Tim Jackson, Marius Jurgilius, Jack Meaning and Sigitas Siaudinis as well as the participants of the CEBRA Annual Conference 2022, the joint CEPR-ECB Conference on The Macroeconomic Implications of Central Bank Digital Currencies and the members of the Eurosystem Monetary Policy Committee Expert Group on CBDC. The bulk of this work was undertaken prior to my employment at the Bank of England and the views expressed in this paper are those of the author and do not necessarily reflect the position of the Bank of England, its committees, the Bank of Lithuania or the Eurosystem.

[†]Bank of England. benjamin.hemingway@bankofengland.co.uk

1 Introduction

The introduction of a retail central bank digital currency (CBDC) is currently under active consideration by central banks around the world. Through the introduction of a retail CBDC, households will be able to hold a bank account directly with the central bank. As a CBDC is likely to have technical features that make it a closer substitute for bank deposits than physical cash. Thus, CBDC is likely to be a greater source of competition for banks in the deposit market. This competition channel has already been extensively studied in the literature, for example, by Andolfatto (2021), Piazzesi and Schneider (2022), and Chiu et al. (2023), among others. In addition to banks facing increased competition for deposits, the introduction of a CBDC is likely to require additional settlement transactions between the banking sector and the central bank, which may further increase bank funding costs. This paper focuses on the consequences of this cost channel for the structure of the deposit market and the implementation of monetary policy.

To this end, I propose a theoretical model in which CBDC and bank deposits are imperfect substitutes and where deposits are subject to liquidity shocks. Banks are able to transfer liquidity between themselves through an interbank market. I assume that it is more costly for banks to trade with the central bank than in the interbank market. This cost could materialize, for example, due to central banks requiring better quality collateral than would be required in the interbank market. In this paper, I assume that the increased costs occur as, following the liquidity shocks, banks are only able to trade with the central bank via standing facilities. As a consequence, the introduction of a CBDC increases the banking sector's use of the central bank standing facilities, and thus increases the costs associated with deposits. In this setting, CBDC raises the cost of bank funding in two ways; directly by competing for depositors and indirectly by increasing the number of transactions with the central bank.

The deposit market is modeled as in the spatial competition model of Salop (1979) with the addition of a central bank. A continuum of atomistic depositors chooses to deposit their funds at one of a finite number of banks or, through a CBDC, at the central bank. This deviates from existing models of CBDC, where households hold portfolios of liquid assets consisting of bank deposits and CBDC. There is some evidence that many households do not hold multiple deposit accounts simultaneously. As part of the UK Competition and Markets Authority investigation into the retail banking market, they commissioned a survey by GfK NOK which found that only 22% of UK households actively used a personal current account at more than one bank, (Moon et al., 2015). Whether households are more willing to hold both bank deposits and CBDC simultaneously is likely to depend on the specific design choices of CBDC.

Using this model, I study the effect of introducing a CBDC on the structure of the deposit market and its implications for monetary policy. In particular, I focus on two parts of the policy debate around CBDC. First, the effectiveness of the CBDC remuneration rate as an additional tool in the monetary policy toolkit, and second, the implications CBDC has on the transmission of the policy rate through the deposit market. To assess the empirical relevance of the theoretical results, I calibrate the model for the Eurozone.

The model makes several predictions that have important policy implications. First, if the banks do not face liquidity risk from deposit financing, then the introduction of a CBDC results in a fall in the market shares of banks in the deposit market and upward pressure on banks to raise deposit rates in the face of greater competition. In the absence of liquidity risk, the bank deposit rate is strictly increasing in the CBDC remuneration rate. Second, the model proposes a novel liquidity risk channel through which the introduction of a CBDC can further increase the costs of banks operating in the deposit market. As deposits are subject to liquidity risk, the model predicts that as the market share of CBDC increases, so does the size of transactions between the banking sector and the central bank. As a consequence, an increase in the CBDC remuneration rate will increase the cost banks face due to this liquidity risk. Thus, in the presence of this liquidity risk, there is additional downward pressure on the bank deposit rate, which may lead to the bank deposit rate not strictly increasing with the CBDC remuneration rate. These results cast doubt on the use of the CBDC remuneration rate as an additional tool for monetary policy.

This paper also highlights the importance of the liquidity risk channel for the transmission of monetary policy more generally. In the absence of liquidity risk, the model predicts that the bank deposit rate increases one-for-one following an increase in the policy rate, even after the introduction of a CBDC. However, if banks face liquidity risk in the deposit market, introducing a CBDC impacts the transmission of the policy rate and there will be imperfect pass-through to the bank deposit rate. This occurs because raising the policy rate also increases the cost banks face when obtaining additional liquidity from the central bank. Furthermore, the impact of monetary policy will now impact the structure of the deposit market and thus monetary policy will impact the deposit rate to differing degrees. This effect is more pronounced if the CBDC is unremunerated.

The model also allows me to study the impact of a CBDC on the structure of the deposit market. As an extension to the baseline case where the number of banks is fixed, I consider a long-run equilibrium where the number of banks adjusts according to a free entry condition. The model suggests that the introduction of a CBDC will decrease bank profitability. Thus if the number of banks is able to adjust, the deposit market becomes more concentrated following the introduction of CBDC and putting downward pressure on bank deposit rates. This provides an additional channel for adjustment, which dampens the effect on deposit rates and further impacts transmission of monetary policy.

This paper is complementary to the growing literature on the policy implications of CBDC. A large literature focuses on financial stability issues; in particular, both Böser and Gersbach (2020) and Fernández-Villaverde et al. (2021) consider the increased risk of bank runs that can occur if bank depositors had access to a CBDC so that they could transfer their deposits in times of financial stress. Both Brunnermeier and Niepelt (2019) and Niepelt (2020) discuss an equivalence result where appropriate transfers from the central bank to the financial system are capable of neutralizing the impact of introducing a CBDC and mitigate the risk of a CBDC-induced bank run. This paper also introduces liquidity risk of deposits; the focus is not on bank runs, but on the costs imposed on banks when they obtain liquidity from a central bank lending facility.

In casting doubt on the use of the CBDC remuneration rate in the monetary toolkit, this paper contributes to the literature on how CBDC should be remunerated. Agur et al. (2022) consider the welfare trade-off for the central bank when choosing a non-interest-bearing versus an interest-bearing CBDC. Barrdear and Kumhof (2022) find that a countercyclical remuneration rate rule for CBDC can contribute to stabilizing the business cycle. Similarly, Bordo (2021) finds that an interest-bearing CBDC may improve the transmission mechanism of monetary policy. On the other hand, Chiu and Davoodalhosseini (2023) find that a non-interest-bearing CBDC increases bank intermediation and thus welfare, while an interest-bearing CBDC results in bank disintermediation and lower welfare. Williamson (2022) studies various implementations of CBDC and shows how an interest-bearing CBDC can increase welfare by competing with private means of payment.

The results of this paper on the possible impact of CBDC on the transmission of monetary policy can be considered alongside a growing literature on the implications of CBDC for monetary policy. A summary of the possible monetary policy implications of CBDC can be found in Bindseil (2019). For example, Keister and Sanches (2023) suggests that, while CBDC can promote efficient exchange, it can also increase funding costs. Meaning et al. (2021) provide a detailed discussion on the monetary transmission mechanism in general, as well as other possible policy implications. Burlon et al. (2022) study the welfare implications of a CBDC and provide a characterization of the welfare-maximizing CBDC policy rules. Kumhof and Noone (2021) discuss the remuneration of CBDC in detail and its possible use for monetary policy. Kumhof and Noone (2021) propose a two-tier remuneration system, while Barrdear and Kumhof (2022) propose a quantity rule and a price rule for CBDC.

This paper is also closely related to the literature on the impact of CBDC on the banking sector. In a macroeconomic framework, Bacchetta and Perazzi (2021) assume a constant elasticity of substitution between a CBDC and a continuum of monopolistically competitive banks. Andolfatto (2021) analyzes the case of a single monopoly bank where CBDC and bank deposits are perfect substitutes, but there is a fixed cost for depositors to switch between the two. Chiu et al. (2023) study a model of Cournot oligopoly with a finite number of banks where banks compete in quantity rather than the remuneration of deposits. CBDC is assumed to be a perfect substitute for bank deposits, and so imposes a minimum remuneration rate on bank deposits. In Piazzesi and Schneider (2022) banks offer both deposits and credit lines and the introduction of CBDC reduces the supply of bank deposits and increases the cost of credit lines. Both Abad et al. (2023) and Lammersdorf et al. (2023) using different frameworks to this paper consider the interaction of CBDC and the interbank market.

This paper is also related to the literature on spatial models of imperfect competition, as the deposit market is based on the classic paper by Salop (1979). Spatial competition models have been widely used to study deposit markets. For example, Chiappori et al. (1995) study the regulation of deposit rates using a Salop circle model of both loans and deposit markets, while Matutes and Vives (1996) study the impact of deposit insurance in a model of spatial competition in the deposit market. Along similar lines Repullo (2004) investigates the effect of capital requirements on bank behavior when imperfect competition in the deposit market is modeled using a Salop circle. Empirical support for spatial models of the deposit market is provided by Park and Pennacchi (2008) and Ho and Ishii (2011), among others. The structure of competition after the introduction of a CBDC is closely related to Salop Circle models with a center such as Bouckaert (2000) and Madden and Pezzino (2011).

Finally, this paper is also related to the literature on interbank markets. In particular, the theoretical treatment of the interbank market in this paper is closest to that of Hauck and Neyer (2014) and Bucher et al. (2020), who both study the operation of an interbank market within the framework of the Eurozone.

The remainder of this paper is organized as follows. Section 2 presents the model. Section 3 goes into further detail on the bank problem. In Section 4 the equilibrium is presented. Section 5 provides comparative statics on both the impact of the CBDC remuneration rate on the bank deposit rate and the implications for the transmission of monetary policy. Section 6 calibrates and provides a quantitative assessment of the model. Section 7 concludes.

2 Model

I consider a model of the retail deposit market with three discrete periods, $t = 1, 2, 3$. The economy consists of three types of agents; risk-neutral banks, a central bank, and a continuum of depositors.

In the first period $t = 1$, banks enter the deposit market, paying a fixed cost $F > 0$. I consider a symmetric equilibrium with a fixed number $N \geq 2$ of ex ante identical banks that enter in $t = 1$.

Banks have access to a technology that yields an exogenously given return R_L on liquidity. Banks must obtain an exogenously given quantity of liquidity $L > 1$ to operate this technology. Banks obtain liquidity in period $t = 2$, either from the central bank or from depositors. The liquidity bank i obtains from the central bank in $t = 2$ is denoted by B_i and is remunerated at the main policy rate R_f where $R_L > R_f$ such that banks earn a higher return from their technology than the policy rate. Bank i sets a deposit rate R_i and obtains a share q_i of the retail deposit market. Deposits are subject to aggregate liquidity risk that is realized in $t = 3$.

At the end of the final period, $t = 3$, banks must return to a liquidity neutral position. To do so, banks may borrow or lend liquidity through the interbank market or the central bank standing facilities. The central bank offers a deposit facility with an interest rate R_{DF} and a lending facility with an interest rate R_{LF}. The central bank charges penalty rates on these standing facilities such that $R_{DF} < R_f < R_{LF}$. Banks can trade liquidity between themselves in an interbank market. Trade in the interbank market takes place at a state-dependent interbank rate R^s_{IB}, where the superscript s denotes the state associated with the realized liquidity risk. The interest rates on the central bank standing facilities define a corridor that sets an upper and lower bound on the interbank rate.

The retail deposit market is modeled as a Salop circle as in Salop (1979). There is a continuum of depositors located around a circle with unit mass. The banks are located equidistant from each other around the circle. A depositor located at a distance $x \geq 0$ from the bank must pay a linear transportation cost $t_B x \geq 0$ to deposit their funds. Banks compete in prices *à la* Bertrand. The interest rate paid on deposits by bank i, denoted as R_i.

The central bank may also enter the deposit market in $t = 1$ by designing and issuing a CBDC. The interest rate paid on CBDC deposits by is denoted as R_{CB} and is set by the central bank. Demand for CBDC is likely to be driven by the specific design of a CBDC. Survey evidence such as that provided by Abramova et al. (2022) found that there was heterogeneity across survey respondents regarding the perceived need for and potential benefits of a CBDC. They also found that the importance of various potential design features, such as offline functionality, varied across user groups. In order to capture preference heterogeneity for CBDC, I assume that each depositor pays a fixed transport cost to obtain CBDC, with this transport cost drawn uniformly from the interval $t_{CB} \in [\underline{t}, \bar{t}]$. The design choices made by the central bank can be captured through the interval $[\underline{t}, \bar{t}]$.

The entry of CBDC into the deposit market, its design, and its remuneration rate are known to all participants in advance. All bank decisions are made in full knowledge of whether they will compete against a CBDC and are conditional on the CBDC remuneration rate R_{CB} and the CBDC design characteristics capture though the interval $[\underline{t}, \bar{t}]$. Deposits are subject to liquidity shocks that occur in the final period ($t = 3$). Every period bank deposits face an outflow of deposits equal to a fraction $\xi \in [0, 1]$ of the deposits held by that institution. With probability $1 - \lambda$, the CBDC is hit by the same liquidity shock and a fraction ξ of CBDC depositors flow out from CBDC. All outflows of deposits are redistributed evenly amongst all deposit-taking institutions proportional to their share of deposits. With probability λ, CBDC faces no outflow of deposits. Absent CBDC, the aggregate liquidity of the banking sector remains unchanged and banks are able to redistribute liquidity between themselves and return to the initial allocation of liquidity through the interbank market. If CBDC has a positive market share, the aggregate liquidity of the banking sector will change depending on whether CBDC faces an outflow of deposits or not. With probability $1 - \lambda$, the outflows of CBDC equal the inflows from deposits and the aggregate liquidity in the banking sector is not affected by the liquidity shock. While with probability λ, there is a net inflow of deposits into CBDC and aggregate liquidity in the banking sector decreases.

The liquidity shock is modeled in such a way that while total deposits remains unchanged, there may be a reallocation of liquidity from the banking sector to the central bank through bank deposits or vice versa. In cases where there is a net surplus or net deficit in liquidity in the banking sector, banks may will to make use of the central bank standing facilities, in addition to the interbank market, in order to return to a neutral liquidity position at the end of $t = 3$. Given this, the presence of CBDC introduces aggregate liquidity risk of deposits and thus cost of deposit finance, for the banking sector. This cost is increasing in the market share of CBDC. Although these liquidity shocks are similar in spirit to those in papers such as Fernández-Villaverde et al. (2021) that focus on the possibility of CBDC generated bank runs, here there is no risk of bank runs. Instead, liquidity risk generates additional costs of deposits for banks. This cost occurs regardless of whether the banking sector has excess liquidity in aggregate or not. The presence of CBDC in conjunction with the liquidity shocks generates volatility in the aggregate liquidity of the banking sector. This in turn means that banks need to increase their use of the central bank standing facilities in at least one of the states.

To summarize the timing of the model, in the first period ($t = 1$), the central bank decides whether to introduce a CBDC and sets its remuneration rate R_{CB}. A fixed number, N, banks enter in $t = 1$ In the second period ($t = 2$), commercial banks compete in the deposit market by setting a deposit rate R_i and obtaining liquidity B_i from the central bank. In the third period ($t = 3$) the liquidity shock is realized and commercial banks use the central bank standing facilities and the interbank market to obtain a liquidity neutral position. In what follows, I focus on the symmetric equilibrium and solve for the Subgame Perfect Nash Equilibrium in pure strategies using backward induction.

3 Banking Sector

In this section I describe the problem faced by the banking sector. As the equilibrium is solved backwards I begin the analysis of the banking sector with the final period, $t = 3$. I then turn to the bank's problem of choosing its funding structure and setting the deposit rates in $t = 2$.

3.1 Bank Liquidity

In the final period, $t = 3$, the $N \geq 2$ banks, indexed by i, have made their decisions about their funding structure. The bank's funding structure consists of a quantity of deposits q_i and central bank liquidity B_i.

The bank's choice of liquidity B_i and deposits q_i implies that before the realization of the liquidity shocks the banks have the following *ex ante* liquidity deficit

$$\epsilon_i \equiv L - B_i - q_i. \tag{1}$$

The *ex ante* aggregate liquidity in the banking sector equal to $\sum_i \epsilon_i$.

With probability $1 - \lambda$, a fraction ξ of all depositors relocate to locations evenly distributed around the Salop circle. Here banks face the same liquidity inflows as liquidity outflows and their ex post liquidity deficit is simply equal to their *ex ante* liquidity deficit. Following the realisation of these liquidity shocks, banks adjust their liquidity holdings through the interbank market rate and central bank standing facilities.

Absent CBDC, these liquidity shocks do not affect aggregate liquidity in the banking sector. If CBDC has a positive market share, then with probability λ, CBDC deposits are not subject to an outflow and thus the aggregate banking sector faces a net outflow of liquidity. In this case, the amount of banking sector liquidity that is reallocated is equal to $(1 - q_{CB})\,\xi$. Of this a fraction q_{CB} flows into CBDC and out of the banking sector. Noting that $\sum_i q_i = 1 - q_{CB}$, the expected *ex post* liquidity deficit faced by bank i in this case can be written as

$$\epsilon_i^+ = \epsilon_i + q_i q_{CB} \xi. \tag{2}$$

With probability $1 - \lambda$, CBDC deposits are also subject to an outflow. As banks face the same liquidity inflows as outflows, bank i's *ex post* liquidity deficit equals their *ex ante* liquidity deficit

$$\epsilon_i^0 = \epsilon_i. \tag{3}$$

It is assumed that banks must return to a liquidity neutral position at the end of $t = 3$. The amount of liquidity they must trade to achieve this depends on the bank's *ex post* liquidity deficit ϵ_i^s, where $s \in \{0, +\}$ denotes the realization of the liquidity shock. If $\epsilon_i^s > 0$, bank i needs to obtain additional liquidity through the interbank market or through the central bank liquidity facility. If, on the other hand, $\epsilon_i^s < 0$ bank i must reduce its liquidity by lending in the interbank market or depositing liquidity in the central bank deposit facility.

The interest rates on the central bank standing facilities act as upper and lower bounds on the interbank rate. Whether these bounds are reached depends on the aggregate liquidity deficit of the banking system $\sum_i \epsilon_i^s$. The relationship between the interbank market and the realization of the aggregate liquidity deficit is set out in the following equation.

$$R_{IB}^s \begin{cases} = R_{LF} & \text{if } \sum_i \epsilon_i^s > 0 \\ = R_{DF} & \text{if } \sum_i \epsilon_i^s < 0 \\ \in [R_{DF}, R_{LF}] & \text{otherwise.} \end{cases} \tag{4}$$

In the case where aggregate liquidity in the banking sector is in surplus, the interbank market rate is equal to the floor rate R_{DF}. Conversely, in the case where the aggregate liquidity in the banking sector is in deficit, the interbank market equals the ceiling rate R_{LF}. Should the banking sector have exactly the amount of liquidity it requires, the interbank market will like within the interest corridor.

The expected liquidity cost of deposits is simply the expected cost of returning to a liquidity neutral position

$$E\left[C_i\right] = (1 - \lambda)\, R_{IB}^0 \epsilon_i^0 + \lambda R_{IB}^+ \epsilon_i^+. \tag{5}$$

By combining equations (2) and (3) with equation (5), the bank's expected cost of deposits can be written as

$$E\left[C_i\right] = \epsilon_i R_f + \lambda \xi R_{IB}^{+} q_{CB} q_i. \tag{6}$$

Given $R_{IB}^{+} \geq R_{IB}^{0}$ it follows from equation (6) that if $\lambda\xi > 0$ and there is aggregate liquidity risk, the expected cost of deposits is increasing in the market share of the central bank. This is an important feedback mechanism of the model. An increase in the market share of CBDC increases the liquidity risk of deposits, and thus increases the expected cost of deposits for banks.

3.2 Bank's Problem

Consider the bank's problem in the intermediate period, $t = 2$, taking the number of banks N as given. In this period, the bank decides on its funding structure by obtaining liquidity from the central bank and sets the interest rate it offers to depositors R_i. Banks are risk neutral and maximize expected profits. The profit function of bank i is

$$\pi_i = \max_{B_i, R_i} \left\{ R_L L - R_f B_i - R_i q_i - E\left[C_i\right] - F \right\}, \tag{7}$$

where $E\left[C_i\right]$ is defined in equation (5) and $F > 0$ is the fixed cost that banks are assume to pay in order to enter the deposit market. Banks compete for depositors in prices à la Bertrand, taking as given both the deposit rates set by other banks and the funding structure of other banks.

If the central bank does not introduce a CBDC, competition between banks in the deposit market is identical to the Salop circle model. If bank i offers a deposit rate equal to R_i and other banks offer a deposit rate equal to R_{-i}, then a depositor located at a distance x from bank i, where $x \in \left[0, \frac{1}{N}\right]$, will choose to deposit their funds at bank i rather than the neighboring bank so long as

$$R_i - t_B x \geq R_{-i} - t_B \left(\frac{1}{N} - x \right), \tag{8}$$

where t_B is the linear transport cost that is incurred by depositors. Bank i thus faces the following demand function

$$q_i = \frac{1}{N} + \frac{1}{t_B} \left(R_i - R_{-1} \right). \tag{9}$$

If the central bank introduces a CBDC, bank i faces competition not only from the two banks that neighbor it, but also from the CBDC. I assume that the central bank sets a fixed remuneration rate R_{CB} and that depositors incur a transport cost t_{CB} if they deposit funds in the CBDC. The transport costs associated with CBDC are assumed to be drawn randomly from a uniform distribution over the interval $[\underline{t}, \bar{t}]$. Thus a depositor located at distance x from bank i would prefer to deposit funds in bank i rather than in the CBDC so long as

$$x \leq \frac{1}{t_B}\left(R_i - R_{CB} + t_{CB}\right). \tag{10}$$

Following the introduction of CBDC, for a depositor to deposit funds in bank i, they must prefer bank i to the CBDC, as well as all other banks and both equations (8) and (10) must be satisfied.

The market share of deposits obtained by bank i depends on the deposit rate that it sets, R_i, relative to the deposit rate set by the neighboring banks, R_{-i} and the CBDC remuneration rate R_{CB}. To characterize the demand function of bank i, it is helpful to define some additional variables.

First, define x_i^* as the distance from bank i where a depositor is indifferent between bank i and the neighboring bank $-i$. The equation for x_i^* follows from equation (8) and is given by

$$x_i^* = \frac{1}{2}\left(\frac{1}{N} + \frac{1}{t_B}\left(R_i - R_{-i}\right)\right). \tag{11}$$

Now consider a depositor with the lowest possible CBDC related cost, $t_{CB} = \underline{t}$. Denote by $\underline{x}_i$ the smallest distance from bank i a depositor with $t_{CB} = \underline{t}$ would prefer depositing their funds into CBDC rather than bank i. From equation (10) this distance can be expressed as

$$\underline{x}_i = \max\left(\frac{1}{t_B}\left(R_i - R_{CB} + \underline{t}\right), 0\right). \tag{12}$$

If CBDC is sufficiently undesirable that, $\underline{x}_i \geq x_i^*$, then any depositor that prefers bank i deposits to those of another bank would also prefer bank i deposits to CBDC. Banks compete for deposits with their neighboring banks. In a symmetric equilibrium where $R_i = R_{-i}$ each bank obtains a market share of $1/N$ as in the standard Salop setup and the demand for deposits follows from (11) as

$$\lim_{\underline{x}_i \to x_i^*} q_i = 2x_i^*. \tag{13}$$

Similarly, consider a depositor with the highest possible CBDC related cost, $t_{CB} = \bar{t}$. Denote by $\bar{x}_i$ the smallest distance from bank i a depositor with $t_{CB} = \bar{t}$ would prefer depositing their funds into CBDC rather than bank i. From equation (10) this distance can be expressed as

$$\bar{x}_i = \min\left(\frac{1}{t_B}\left(R_i - R_{CB} + \bar{t}\right), x_i^*\right). \tag{14}$$

If CBDC is sufficiently desirable that, $\bar{x}_i \leq 0$, then no depositor would prefer bank i deposits to CBDC and thus

$$\lim_{\bar{x}_i \to 0} q_i = 0. \tag{15}$$

For depositors located within the interval $[\underline{x}_i, \bar{x}_i]$ from bank i, there exists some value of t_{CB} such that depositors would be indifferent between depositing their funds into CBDC or at bank i. Denote by $t_i^*(x)$ the collection of values of t_{CB} for which this holds where

$$t_i^*(x) = R_{CB} - R_i + t_B x. \tag{16}$$

The demand function facing bank i is thus

$$q_i = 2\left(\int_{\underline{x}_i}^{\bar{x}_i}\left(\frac{t_B - t_i^*(x)}{\bar{t} - \underline{t}}\right)dx + \underline{x}_i\right). \tag{17}$$

As depositors are assumed not to have an outside option, there will be full coverage in the deposit market and all deposits will be deposited either at a retail bank or at the central bank. Thus the market share of CBDC can be written as

$$q_{CB} = 1 - \sum_i q_i. \tag{18}$$

4 Equilibrium

I focus on solving for a symmetric equilibrium in which all banks make identical decisions about their funding structure: B_i and set the same deposit rate R_i. As banks set the same deposit rate, they obtain an equal share of deposits q_i. I focus on the case where the number of banks N is fixed.

4.1 Interbank Market and Bank Funding Structure

I begin by characterizing the equilibrium funding structure of the bank chosen in $t = 2$ and the equilibrium interest rate of the interbank market in $t = 3$. In choosing their funding structure, banks take the interest rates in the interbank market, the policy rate and the interest rates on standing facilities as given. Conditional on the deposit rate they set, banks perfectly anticipate the market share of deposits they obtain. Obtaining one additional unit of liquidity from the central bank in $t = 2$ has a marginal cost of R_f, while also reducing by one unit the bank's *ex ante* liquidity deficit ϵ_i. Therefore, in equilibrium, banks adjust B_i so that the marginal cost of increasing B_i, R_i, is equal to the expected marginal cost of increasing its *ex ante* liquidity deficit ϵ_i and the following condition holds

$$\frac{\partial E\left[C_i\right]}{\partial \epsilon_i} = R_f. \tag{19}$$

In equilibrium, the interbank rates that hold in $t = 3$ must be consistent with the bank's funding decisions made in $t = 2$. Given a bank's choice of B_i and its market share q_i, a bank's *ex post* liquidity deficit ϵ_i^s is conditional on the realization of the liquidity shock $s \in \{0, +\}$. In equilibrium, the interbank rate conditional on liquidity shock s can be found from equation (4). The equilibrium interbank rate and the bank's equilibrium funding structure are summarized in Proposition 1 below.

Proposition 1. *In equilibrium, banks obtain liquidity B_i from the central bank in $t = 2$ such that*

- **I.** *When $\lambda \leq \left(\frac{R_f - R_{DF}}{R_{LF} - R_{DF}}\right)$, $B_i = L - q_i$. The equilibrium interbank market rates are $R_{IB}^0 = R_{LF} - \left(\frac{1}{1-\lambda}\right)\left(R_{LF} - R_f\right)$ and $R_{IB}^+ = R_{LF}$.*
- **II.** *When $\lambda > \left(\frac{R_f - R_{DF}}{R_{LF} - R_{DF}}\right)$, $B_i = L - \left(1 - q_{CB}\xi\right) q_i$. The equilibrium interbank market rates are $R_{IB}^0 = R_{DF}$ and $R_{IB}^+ = R_{DF} + \frac{1}{\lambda}\left(R_f - R_{DF}\right)$.*

Proof. See the Appendix. □

An implication of Proposition 1 is that interest rates in the interbank market depend on λ, the probability that the banking sector is hit by a net liquidity outflow. If λ is low, then banks have a neutral liquidity position if the banking sector is not hit by an outflow of liquidity, $\epsilon_i^0 = 0$. However, if λ is sufficiently high, then banks have a neutral liquidity position if the banking sector is hit by an outflow of liquidity and hold surplus liquidity otherwise, $\epsilon_i^+ = 0$. As the probability of being hit by a liquidity shock increases, banks have a greater incentive to accumulate liquidity in $t = 2$, and thus the supply of liquidity in the banking sector in $t = 3$ increases. As a consequence, conditional interbank rates are weakly decreasing in λ.

4.2 Deposit Market Equilibrium without CBDC

I now turn to the equilibrium in the deposit market. In $t = 2$ bank i sets a deposit rate R_i that in combination with its funding decision set out in Proposition 1 maximizes its expected profit.

The equilibrium deposit rate can be found by differentiating the bank's profit function given by equation (7) with respect to the deposit rate chosen by the bank, R_i yielding

$$-q_i - \frac{\partial q_i}{\partial R_i}\left(R_i + \frac{\partial E\left[C_i\right]}{\partial q_i}\right) - \frac{\partial q_{CB}}{\partial R_i}\frac{\partial E\left[C_i\right]}{\partial q_{CB}} = 0. \tag{20}$$

First, I focus on the case where CBDC does not have a share of the deposit market, $q_{CB} = 0$. Absent CBDC, equation (20) simplifies to

$$R_i = R_f - \frac{1}{N}t_B, \tag{21}$$

where banks set deposit rates lower than their cost of funds R_f according to their market power, which depends on both the number of banks, N and preference heterogeneity for deposits which is determined by t_B.

For CBDC to have no market share in equilibrium, the depositors that value CBDC the most, with cost $t_{CB} = \underline{t}$ of holding CBDC, would still prefer to deposit in a bank, and thus $\underline{x}_i \geq x_i^*$. By combining equations (11) and (12) with the equilibrium interest rate absent CBDC in equation (21), it follows that CBDC has no market share if

$$R_{CB} \leq R_f - \frac{3}{2N}t_B + \underline{t} \equiv \underline{R}_{CB}. \tag{22}$$

As depositors are assumed to have no outside option, with $R_{CB} \leq \underline{R}_{CB}$ depositors will always choose to deposit their funds in a bank. In the symmetric equilibrium studied here, banks obtain equal market shares, and thus bank i's market share is $q_i = \frac{1}{N}$. The equilibrium without CBDC is characterized by Proposition 2.

Proposition 2. *If the number of banks is fixed at $N \geq 2$ and $R_{CB} \leq \underline{R}_{CB}$ then there exists a unique symmetric equilibrium where banks compete such that every bank i sets the same deposit rate $R_i = R_f - \frac{1}{N}t_B$ and obtains the same share of deposits $q_i = \frac{1}{N}$. CBDC has zero market share $q_{CB} = 0$.*

Proof. See the Appendix. □

With $q_{CB} = 0$, the bank's equilibrium funding decision combined with equation (6) implies that $E\left[C_i\right] = 0$ and thus the expected liquidity cost of deposits is zero. This is a direct consequence of the structure of the liquidity shock. In an economy without CBDC, banks face net inflows of liquidity that exactly offset the net outflows of liquidity regardless of the realization of the aggregate liquidity shock, s. It is optimal for banks to accumulate liquidity so that they do not need to make use of either of the central bank's standing facilities.

The profit bank i makes in the case where $R_{CB} \leq \underline{R}_{CB}$ can be found by substituting the equilibrium deposit rate and market share into equation (7), yielding

$$\pi_i = \bar{\pi} - \left(R_i - R_f + \lambda\xi R_{IB}^{+} q_{CB}\right) q_i, \tag{23}$$

where

$$\bar{\pi} \equiv \left(R_L - R_f\right) L - F. \tag{24}$$

4.3 Deposit Market Equilibrium with CBDC

Now, consider the bank's choice of deposit rate when $R_{CB} > \underline{R}_{CB}$, and thus the CBDC remuneration rate is sufficiently high that it poses meaningful competition to banks. With $q_{CB} > 0$, the market share of each bank in a symmetric equilibrium is no longer equal to $\frac{1}{N}$ and instead depends on the deposit rate offered by the banks. As a consequence, the deposit rate is now determined by the first-order condition for the deposit rate, equation (20), and the definitions of x_i^*, $\underline{x}_i$ and $\bar{x}_i$ set out by equations (11), (12) and (14).

For CBDC to dominate the market and drive banks from the deposit market, it must be preferred to all banks by all depositors. Depositors that have the strongest preference for bank i with $x_i = 0$ and the weakest preference for CBDC with $t_{CB} = \bar{t}$ strictly prefer the CBDC to bank i deposits so long as $R_{CB} > R_i + \bar{t}$. This is more likely if the CBDC offers a sufficiently high remuneration rate relative to bank deposits and if the design of the CBDC is well received by depositors such that $\bar{t}$ is low. In the Appendix, I show that in equilibrium, a CBDC will dominate the market if the following condition is satisfied:

$$R_{CB} > R_f - \frac{1}{2}\lambda\xi R_{IB}^{+} + \bar{t} \equiv \bar{R}_{CB}. \tag{25}$$

To simplify the analysis, I focus on the case where the liquidity cost facing banks is not so large and the demand for CBDC so homogeneous that banks are forced out of the deposit market almost immediately. Specifically, I assume that the following parameter restriction holds

$$\lambda\xi R_{IB}^{+} \leq \frac{3}{2}\left(\bar{t} - \underline{t}\right). \tag{26}$$

In a symmetric equilibrium where the number of banks is fixed at N, all banks set identical deposit rates. Equation (11) simplifies to $x_i^* = \frac{1}{2N}$. If $R_{CB} > \underline{R}_{CB}$, the CBDC remuneration rate is sufficiently high that $q_{CB} > 0$. In this case, by combining equations (14) and (20) the equilibrium can be found as the $\bar{x}_i$ that solves the following equation

$$\Gamma \equiv -q_i + (R_f - R_{CB} - t_B\bar{x} + \bar{t})\frac{\partial q_i}{\partial R_i} - \lambda\xi R_{IB}^{+}\left(q_{CB}\frac{\partial q_i}{\partial R_i} + q_i\frac{\partial q_{CB}}{\partial R_i}\right) = 0, \qquad (27)$$

where q_i and q_{CB} are functions of x_i^* and $\bar{x}_i$ and given by equations (17) and (18), respectively.

The bank takes the deposit rates set by the other banks, as well as the CBDC remuneration rate, as given. It chooses its deposit rate R_i taking into account the effect that a change in the deposit rate has on both its own market share, q_i, and on the market share of CBDC, q_{CB}.

Equation (27) also depends on $\frac{\partial q_{CB}}{\partial R_i}$ which is the impact of an increase in R_i on the market share of CBDC, holding the deposit rates of other banks fixed. This can be obtained through the definition of q_{CB} set out by equation (18). As depositors do not have an outside option, they must deposit their funds at a bank or in the CBDC. Therefore, the market share of CBDC is the mass of depositors who choose not to deposit funds at any bank. An increase in the deposit rate set by bank i, R_i, affects the market share not only of bank i, but also of neighboring banks; the impact of R_i on the market share of CBDC can be calculated from

$$\frac{\partial q_{CB}}{\partial R_i} = -\frac{\partial q_i}{\partial R_i} - \frac{\partial q_{i+1}}{\partial R_i} - \frac{\partial q_{i-1}}{\partial R_i}, \qquad (28)$$

where q_{i+1} and q_{i-1} denote the market shares of neighboring banks.

The equilibrium is summarized by the $\bar{x}_i$ that solves equation (27). In cases where the CBDC remuneration rate is sufficiently high that $R_{CB} > \underline{R}_{CB}$, it follows from equation (27) that $\underline{x}_i < x_i^*$ and CBDC obtains a positive share of the deposit market, $q_{CB} > 0$.

For a given CBDC design, $(\underline{t}, \bar{t})$ there exists a threshold remuneration rate R_{CB} such that $\bar{x}_i < x_i^*$, found from equations (11) and (14), above which banks do not directly compete with each other. Instead, banks operate a local monopoly in which they compete only with the CBDC for depositors.

Should the CBDC remuneration rate increase above some upper limit such that $R_{CB} > \bar{R}_{CB}$, then it follows from equation (14) that banks will not operate in the deposit market and all depositors hold CBDC. These results are summarized in the following proposition.

Proposition 3. *Given that $\lambda\xi R_{IB}^{+} \leq \frac{3}{2}(\bar{t} - \underline{t})$ and the number of banks is fixed at $N \geq 2$ then if $R_{CB} > \underline{R}_{CB}$*

- **I.** *When $R_{CB} \leq \bar{R}_{CB}$ some depositors hold CBDC, $q_{CB} \in (0, 1)$. For a given CBDC design, $(\underline{t}, \bar{t})$, the market share of banks, q_i is strictly decreasing in R_{CB} while the market share of CBDC is strictly increasing in R_{CB}.*

II. *When $R_{CB} > \bar{R}_{CB}$ banks do not operate in the deposit market and CBDC dominates such that $q_{CB} = 1$.*

Proof. See the Appendix. □

In an equilibrium with a fixed number of banks, the market share of banks is strictly decreasing in R_{CB} over the interval $\left(\underline{R}_{CB}, \bar{R}_{CB}\right]$. As the number of banks is fixed, this also results in an increase in the market share of CBDC. A higher remuneration rate of CBDC leads to more depositors choosing CBDC over bank deposits.

Although the central bank balance sheet is not explicitly modeled, Proposition 1 highlights that in equilibrium, each bank increases its holdings of central bank liquidity (B_i) as its market share decreases. Summing over all N banks and using the definition of q_{CB} given in equation (18) yields the following equation for the aggregate liquidity borrowed from the central bank by the banking sector

$$\sum_i B_i = NL - (1 - q_{CB}). \tag{29}$$

Thus, as the market share of CBDC increases, the aggregate banking sector holds more central bank liquidity, and therefore the introduction of a CBDC increases both the liabilities (q_{CB}) and the assets ($\sum_i B_i$) on the central bank balance sheet.

In the case where $q_{CB} > 0$, the profit a bank makes by setting the deposit rate at the profit maximizing level can be written as a function of $\bar{x}_i$ and x_i^*. Substituting equation (14) into (23) and noting that both q_{CB} and q_i will be functions of $\bar{x}_i$ and x_i^* in equilibrium yields

$$\pi_i = \bar{\pi} - \left(R_{CB} - R_f + t_B\bar{x} - \bar{t} + \lambda\xi R_{IB}^{+} q_{CB}\right) q_i. \tag{30}$$

5 Comparative Statics

5.1 Impact of CBDC on deposit rates

I now present the impact of a change in the CBDC remuneration rate, R_{CB}, on the equilibrium deposit rate R_i offered by the banks. The impact of the CBDC remuneration rate depends on whether the bank faces liquidity costs in its use of deposits. I consider the impact of a change in R_{CB} in two liquidity scenarios. First, when the expected size of the liquidity shock is zero, $\lambda\xi = 0$, and banks do not face a liquidity cost. Second, consider the case where the size of the liquidity shock is positive, $\lambda\xi > 0$, and banks face a liquidity cost to hold deposits that increases in the market share of CBDC. The results presented in this section are especially relevant to the policy question of whether the remuneration rate of a CBDC can be used as an additional tool in the central bank's toolbox, as has been discussed among others in Meaning et al. (2021).

In the case where $R_{CB} \leq \underline{R}_{CB}$ and $q_{CB} = 0$, CBDC has no market share and the deposit rate is given by Proposition 2. An increase in the CBDC remuneration rate will not have an impact on the bank deposit rate. This holds regardless of the value $\lambda\xi$ takes.

If $R_{CB} > \underline{R}_{CB}$ so that in an equilibrium CBDC has a positive market share, $q_{CB} > 0$, the impact of an increase in R_{CB} on the deposit rate in the equilibrium can be found by using the implicit function Theorem and rearranging equation (14) and differentiating with respect to R_{CB} which yields

$$\frac{\partial R_i}{\partial R_{CB}} = 1 + t_B \left.\frac{d\bar{x}_i}{dR_{CB}}\right|_{\Gamma=0}. \tag{31}$$

In general, the pass-through of an increase in R_{CB} to the deposit rate will be imperfect since an increase in the CBDC remuneration rate would not lead to an equal increase in the bank deposit rate, $\frac{\partial R_i}{\partial R_{CB}} < 1$. In an equilibrium with $\lambda\xi = 0$, the deposit rate will be strictly increasing in the CBDC remuneration rate, R_{CB}, for all $R_{CB} \in \left[\underline{R}_{CB}, \bar{R}_{CB}\right]$. An increase in the CBDC remuneration rate will result in banks losing market share to CBDC and banks will raise their deposit rates in response to this additional competition.

If the expected value of the liquidity shock is positive $\lambda\xi > 0$, then an increase in the market share of CBDC results in banks facing a higher expected liquidity cost from holding deposits. Therefore, an increase in the CBDC remuneration rate, R_{CB}, not only increases the competition faced by banks in the deposit market, but also increases the liquidity cost of deposits, making deposits a less desirable form of liquidity for banks to hold. As a consequence of this, the pass-through of the CBDC remuneration rate to the deposit rate is lower than it would be in the case without liquidity shocks $\lambda\xi = 0$. Furthermore, if deposits become less desirable for banks to hold, they may prefer to switch to other sources of liquidity rather than raise deposit rates to compete for additional market share. As a result in the presence of liquidity risk, the deposit rate is not guaranteed to increase in the CBDC remuneration rate.

These results are summarized in the following proposition.

Proposition 4. *Given that $\lambda\xi R^{+}_{IB} \leq \frac{3}{2}\left(\bar{t} - \underline{t}\right)$ and the number of banks is fixed at $N \geq 2$ then in equilibrium*

- **I.** *If $q_i > 0$, the pass-through of the CBDC rate to the deposit rate is imperfect $\left(\frac{\partial R_i}{\partial R_{CB}} < 1\right)$.*

II. *The pass-through is positive for any R_{CB} larger but sufficiently close to $\underline{R}_{CB}$.* $\left(\lim_{R_{CB}\downarrow \underline{R}_{CB}} \left\{\frac{\partial R_i}{\partial R_{CB}}\right\} > 0\right)$

III. *If $\lambda\xi R_{IB}^{+} = 0$ the pass-through is positive* $\left(\frac{\partial R_i}{\partial R_{CB}} > 0\right)$ *for all* $R_{CB} \in \left[\underline{R}_{CB}, \bar{R}_{CB}\right]$.

Proof. See the Appendix. □

The results set out in Proposition 4 have important policy implications. In particular, regarding the use of the CBDC remuneration rate as an additional tool in the central bank's toolkit. Even in the most benign scenario where the number of banks is fixed and there is no liquidity shock, the pass-through of the CBDC remuneration rate to the bank deposit rate is imperfect. In this scenario, while banks raise their deposit rates in response to increased competition from CBDC, as a consequence of the imperfect competition in the deposit market, do so less than one-for-one. Finally, in the case where there is risk of liquidity flowing from bank deposits to a CBDC, the additional cost this imposes on banks further weakens the pass-through of the CBDC remuneration rate to the bank deposit rate, and in some cases an increase in the CBDC remuneration rate may result in a fall in the bank deposit rate.

5.2 Monetary Policy Transmission

In this section, I consider the implications of CBDC for the transmission of monetary policy within the context of the model. To this end, I add some additional structure to the model in the following way. First, I assume that the spreads on the central bank standing facilities are held fixed and that the interest rate on the liquidity facility and on the deposit facility are of the form

$$R_{LF} = R_f + \Delta_{LF}, \tag{32}$$

and

$$R_{DF} = R_f - \Delta_{DF}, \tag{33}$$

with $\Delta_{LF} > 0$ and $\Delta_{DF} > 0$. Second, I assume that the interest rate on bank loans is equal to the policy rate plus a fixed mark-up such that

$$R_L = R_f + \Delta_L, \tag{34}$$

with $\Delta_L > 0$. Finally, I assume that the central bank sets the remuneration rate of CBDC such that there is a fixed spread between the remuneration rate and the policy rate such that

$$R_{CB} = R_f + \Delta_{CB}. \tag{35}$$

Here, Δ_{CB} could be positive or negative, depending on the remuneration rate of CBDC. It should also be noted that this is just one possible remuneration policy that central banks could choose for CBDC. However, as will be shown later when I consider the case of an unremunerated CBDC, the remuneration policy considered here is the most neutral implementation of CBDC remuneration in the model. Other remuneration policies can be obtained by combining a change in the policy rate with a change in R_{CB}.

In the case where CBDC has no market share ($q_{CB} = 0$) it follows from Proposition 2 that the deposit rate increases one-for-one with the policy rate. The most interesting case occurs when CBDC has a share of the deposit market ($q_{CB} > 0$). Given the above assumptions on interest rates, the key equation that determine the equilibrium, equation (27), can be rewritten as follows

$$\tilde{\Gamma} \equiv -q_i - (\Delta_{CB} - \bar{t} + t_B\bar{x})\frac{\partial q_i}{\partial R_i} - \lambda\xi\left(R_f + \Delta_{IB}^{+}\right)\left(q_{CB}\frac{\partial q_i}{\partial R_i} + q_i\frac{\partial q_{CB}}{\partial R_i}\right) = 0, \tag{36}$$

where

$$\Delta_{IB}^{+} = \begin{cases} \Delta_{LF} & \text{if } R_{IB}^{+} = R_{LF} \\ \left(\frac{1-\lambda}{\lambda}\right)\Delta_{DF} & \text{if } R_{IB}^{+} = R_{DF} + \frac{1}{\lambda}\left(R_f - R_{DF}\right). \end{cases} \tag{37}$$

In the case where $q_{CB} > 0$, the CBDC remuneration rate is sufficiently high that $R_{CB} > \underline{R}_{CB}$ and the deposit rate can be written in terms of R_f as

$$R_i = t_B\bar{x}_i + R_f + \Delta_{CB}. \tag{38}$$

From equation (38) it follows that if $\bar{x}_i$ and thus the market share of CBDC is held fixed then the deposit rate moves one-for-one with the policy rate. Furthermore if banks do not face liquidity risk, $\lambda\xi = 0$, then banks can pass on the increase in the policy rate to depositors without affecting their market shares and the deposit rate will move one-for-one with the policy rate.

If, on the other hand, banks face liquidity risk and $\lambda\xi > 0$, an increase in the policy rate increases the liquidity cost of deposits that banks face and banks are unable to maintain the status quo by passing the policy rate change onto depositors. From equation (36) it follows that an increase in the policy rate R_f now affects $\bar{x}_i$ and hence the market share of CBDC.

This result is summarized in the following proposition.

Proposition 5. *Given the policy rates defined in equations* (32)*,* (33)*,* (34)*, and* (35) *if* $\lambda\xi > 0$ *then for any equilibrium that features* $q_i > 0$*, the pass-through of the policy rate to the deposit rate is imperfect* $\left(\frac{\partial R_i}{\partial R_f} \neq 1\right)$.

Proof. See the Appendix. □

The key mechanism driving the imperfect pass-through of the policy rate is the liquidity cost of deposits described in equation (6). If banks face liquidity risk $\lambda\xi > 0$, an increase in the policy rate also increases the cost of obtaining additional liquidity should the bank require it. As deposits become less desirable for the bank to hold, there is downward pressure on bank deposit rates, and banks require larger spreads to compensate for the additional liquidity risk.

Proposition 5 highlights a possible risk that the introduction of CBDC poses to the transmission of monetary policy to the economy. In the model, monetary policy transmission occurs solely through pass-through of the policy rate to the deposit rate set by banks. In the case without deposit liquidity risk, a CBDC can be introduced without impacting this transmission channel. However, if banks face a liquidity risk in obtaining liquidity from retail deposits, this cost will increase in the deposit rate and, in turn, will affect the transmission of monetary policy through the deposit rate. This occurs because the cost of this liquidity risk that banks face depends on the cost of obtaining additional liquidity through the central bank lending facility. The cost of obtaining this liquidity increases with the policy rate.

6 Quantitative Analysis

The previous section provides some theoretical results on the impact of a CBDC on both the deposit rate and the transmission of monetary policy. To address whether these theoretical results are quantitatively important, I present a simple calibration of the model to the Eurozone economy without CBDC. I also consider the consequence of an unremunerated CBDC for monetary policy transmission.

6.1 Calibration

Data are obtained from the ECB Statistical Data Warehouse. The data obtained are averaged over the year 2021, which is the last year data are available for all of the series. The policy rates in the model are calibrated to the corresponding ECB rates. The main policy rate R_f is calibrated to the ECB's Main Refinancing Rate which was 0 throughout 2021. The interest rates on the standing facilities, R_{LF} and R_{DF} are calibrated to the ECB's Lending Facility Rate and Deposit Facility Rate which were 25 basis points and -50 basis points, respectively.

The parameters affecting the banking sector are calibrated so that in equilibrium there is no CBDC ($q_{CB} = 0$) and the free-entry condition holds. The number of banks in this equilibrium is set to $N = 7$. This is chosen to match the Herfindahl Hirschman Index (HHI) of Eurozone credit institutions which averaged 0.145 in 2021. Given that the model assumes banks of equal size, the HHI corresponds to $1/N$.

Given $q_{CB} = 0$, the equilibrium deposit rate is given by Proposition 2 as

$$R_i = R_f - \frac{1}{N} t_B. \tag{39}$$

Therefore, the transport cost t_B can be set such that the equilibrium deposit rate R_i matches the average deposit rate in the Eurozone, which was -1.44 basis points. This deposit rate is calculated as the weighted average deposit rate on overnight household deposits and overnight corporate deposits.

In an equilibrium without CBDC, the model predicts that banks hold sufficient liquidity that they do not require additional liquidity from central bank standing facilities or the interbank market. As a consequence, the following equation holds $L = B_i + q_i$. As the size of the banks is normalized by $q_i = 1/N$, L and B_i are calibrated such that the ratio of deposits to total liabilities (q_i/L) in the model matches the ratio of deposit to liabilities of Eurozone credit institutions which in the data is 0.42. The bank lending rate R_L is chosen to match the interest rate on short-term loans to non-financial corporations, which stood at 150 basis points. Finally, the size of the liquidity shock ξ is set to match the percentage of total deposit liabilities that are traded daily in the Target 2 market which in 2021 was 3.13%. The calibration is summarized in Table 1.

6.2 Impact of the CBDC remuneration rate

Using the benchmark calibration, I now plot how the deposit rate R_i changes as R_{CB} varies between $\underline{R}_{CB}$ and $\bar{R}_{CB}$. Figure 1 plots the spread between the deposit rate and the policy rate as the CBDC remuneration rate for various values of λ, holding the number of banks fixed.

Figure 1 illustrates the relationship between the CBDC remuneration rate, R_{CB} and the deposit rate R_i for varying values of λ. The first key takeaway from the graph is that

Table 1: Calibrated Parameters

Parameter	Notation	Value	Calibration Target
Main Policy Rate	R_f	1.0	ECB Main Refinancing Rate
Lending Facility Rate	R_{LF}	1.0025	ECB Lending Facility Rate
Deposit Facility Rate	R_{DF}	0.995	ECB Deposit Facility Rate
Number of Banks	N	7	Herfindahl Hirschman Index (HHI) of Eurozone credit institutions
Bank Lending Rate	R_L	1.015	Interest rate on short-term loans to non-financial corporations
Bank Deposit Rate	R_i	0.99986	Overnight deposit rate of household and corporate deposits
Deposit to Liability Ratio	q_i/L	0.42	Ratio of Deposit to Liabilities of Eurozone Credit institutions
Size of Liquidity Shock	ξ	0.00313	Ratio of Euro short-term rate volume to Eurozone Credit institutions Deposits

the sensitivity of the deposit rate to changes in the remuneration rate is always less than one. This suggests that the use of a CBDC remuneration rate may not be an effective addition to the monetary policy toolbox as increases in the CBDC remuneration rate are only partially transmitted to deposit rates.

In a particular region of Figure 1, where the CBDC remuneration rate is close to but above the threshold level $\underline{R}_{CB}$, the deposit rate is strictly increasing in the CBDC remuneration rate R_{CB} regardless of the value of λ. Here, CBDC poses meaningful competition for banks in the deposit market and banks respond by increasing the deposit rate in an attempt to maintain their market share.

In the case where $\lambda = 0$ and the banks do not face liquidity risk, the deposit rate is strictly increasing in the CBDC remuneration rate R_{CB}. This highlights that absent liquidity risk, the sole response of banks to the increase in competition following a rise in R_{CB} is to compete for market share by increasing their deposit rates.

As λ and thus the liquidity risk facing banks increases, the relationship between R_{CB} and the deposit rate shifts. If the CBDC remuneration rate is sufficiently high, the deposit rate begins to decrease as R_{CB} rises. This occurs because when $\lambda > 0$, an increase in

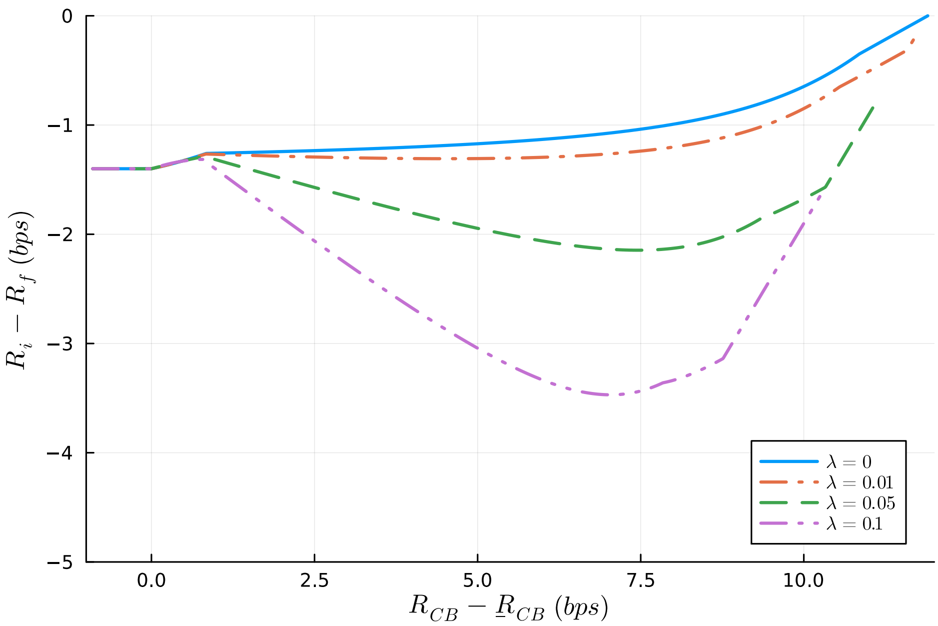

Figure 1: Impact of R_{CB} on the deposit rate

R_{CB} increases both the competition banks face in the deposit market and the liquidity risk banks face. This liquidity risk is increasing in both the probability of being hit by the shock λ, and in the market share of CBDC. When R_{CB} is sufficiently high and thus the market share of CBDC is sufficiently large, the liquidity risk channel dominates the competition channel and banks must increase their deposit rates to compensate for the increased liquidity risk they face. In this case, banks lose additional market share and obtain a larger share of their liquidity from non-deposit financing.

6.3 Implications for Monetary Policy Pass-through

I now plot how the pass-through of the policy rate R_f to the deposit rate is affected by the market share of CBDC. Figure 2 shows the change in the equilibrium deposit rate of to a 10bps increase in the policy rate R_f for differing market shares of CBDC. In this quantitative exercise, the calibration remains as set out in Table 1 while the initial R_{CB} is assumed to vary over the interval $\left[\underline{R}_{CB}, \bar{R}_{CB}\right]$ such that the market share of CBDC varies over the interval $q_{CB} \in [0, 1]$. As in section 5.2, it is assumed that there is a fixed spread between the policy rate and the facility rates, the loan rate and the CBDC remuneration rate. In particular, it is assumed that the CBDC remuneration rate increases in line with the policy rate as specified by equation (35).

Absent liquidity risk when $\lambda = 0$, Figure 2 shows that there is complete pass-through of the policy rate to the deposit rate, regardless of the market share of CBDC. This illustrates the mechanism set out in section 5.2.

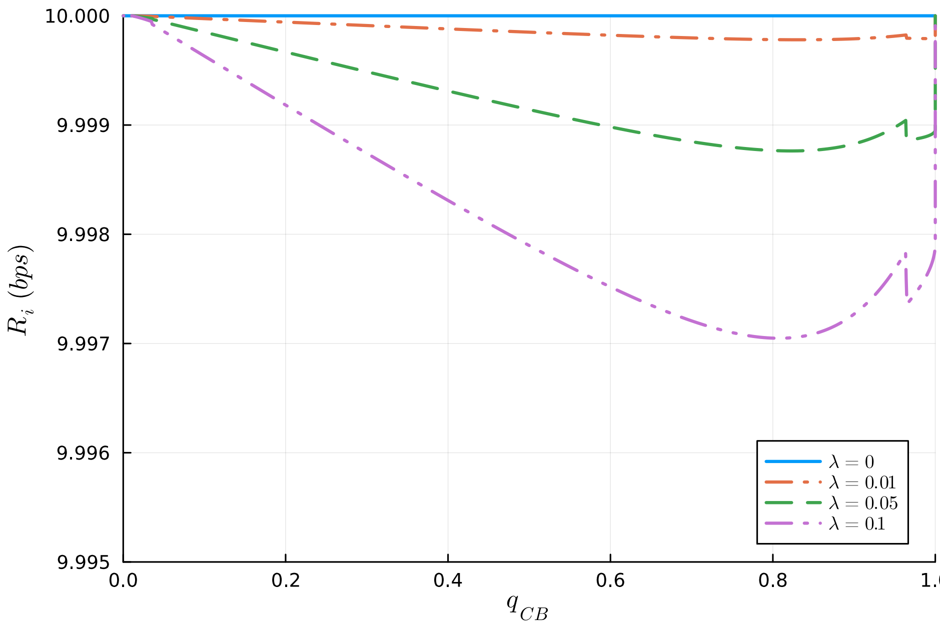

Figure 2: Impact of CBDC on Monetary Policy Transmission

Of more interest, in cases where there is liquidity risk and $\lambda > 0$. Here the pass-through level is imperfect with the increase in the deposit rate lower than the increase in the policy rate. As λ and the liquidity risk increases, the pass-through for a given CBDC market share falls. Here, an increase in the policy rate also increases the liquidity risk that banks face. In order to compensate themselves for the increase liquidity risk of deposits, banks choose not to pass on all of the increased policy rate to depositors and accept a lower market share.

Finally, it is clear from Figure 2 that while $\lambda > 0$ results in imperfect pass-through, the magnitude of this distortion for the baseline calibration is relatively small. Thus while the numerical analysis confirms the theoretical result stated in Proposition 5, it also suggests that, at least with the current parameterization, the magnitude of this effect may not be very large. To understand why, equation (36) shows that an important determinant of the pass-through distortion is the expected size of the liquidity shock, $\lambda\xi$. The values of λ and ξ in the benchmark calibration are quite modest and are the key to driving the low magnitude of the distortion exhibited in Figure 2.

6.4 Unremunerated CBDC

Next I turn to the case of an unremunerated CBDC where $R_{CB} = 1$ is held constant. Figure 3 shows the change in the equilibrium deposit rate of to a 10bps increase in the policy rate R_f for differing market shares of CBDC while holding the CBDC remuneration rate fixed.

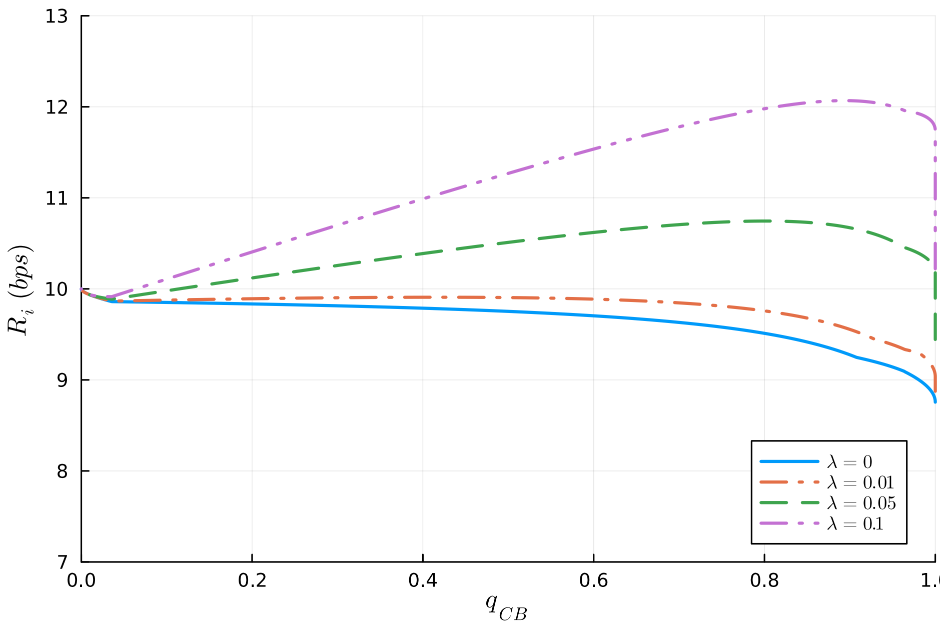

Figure 3: Impact of Unremunerated CBDC on Monetary Policy Transmission

In this quantitative exercise, the calibration for the point $q_{CB} = 0$ remains as set out in Table 1. With $R_{CB} = 1$ fixed, I decrease the initial R_f from the baseline calibration value in order to vary the market share of CBDC over the interval $q_{CB} \in [0, 1]$. The remaining policy rates are assumed to maintain a fixed spread between the policy rate as set out in section 5.2.

First note that as in the previous example, if the CBDC has zero market share, $q_{CB} = 0$, there is full pass-through of the 10bps increase in the policy rate to the deposit rate. This holds for all values of λ as absent CBDC, the structure of the liquidity shock is such that banks face no liquidity risk.

As the market share of CBDC increases, Figure 3 shows that there is no longer one-for-one transmission of monetary policy to the deposit rate. The response of the deposit rate depends on both the market share of CBDC, q_{CB}, and on the liquidity shock, λ. In the case where $\lambda = 0$, indicating banks face no liquidity risk, the pass-through is strictly less than one-for-one for $q_{CB} > 0$. With an unremunerated CBDC, an increase in the policy rate results in a larger spread between the policy rate and the return on CBDC. As a consequence, banks face less competition from the CBDC and absent any liquidity risk are able to increase the spread between the policy rate and the rate they offer depositors.

As λ increases and banks face more liquidity risk, the deposit rate becomes more sensitive to changes in the policy rate. This is a consequence of the fall in CBDC market share that follows an increase in the policy rate while the return on the unremunerated CBDC is fixed at 1. When banks face liquidity risk, the fall in CBDC market share has two effects. First, banks face less competition from CBDC; this puts downward pressure on the pass-through to deposit rates. Second, banks face less liquidity risk as the expected outflows from the banking sector are smaller; this puts upward pressure on deposit rates as the required liquidity risk premium falls.

Figure 3 highlights the interaction of monetary policy pass-through and CBDC market share in two important ways. First, in the region where q_{CB} is positive but close to zero, where CBDC holdings are minimal, the deposit rate increase is slightly lower than the increase in the policy rate. This holds even for relatively large values of λ as when q_{CB} is small, the liquidity risk faced by banks is small due to the low the expected outflows to CBDC. Here the competition channel dominates and banks do not pass through the full increase in R_f to depositors. Second, for sufficiently high values of λ, the liquidity risk channel begins to dominate the competition channel when CBDC has sufficiently high market share. In these cases, the deposit rate increases by more than the policy rate.

6.5 Short-run versus long-run effects

The modeling framework I use also allows me to distinguish between a short-run equilibrium, where the number of banks is held fixed, versus a long-run equilibrium, where the number of banks, N, adjusts according to the free-entry condition. The long-run equilibrium in the deposit market can be summarized as the pair $\{x_i^*, \bar{x}_i\}$ that satisfies equations (27) and (30).

In this section I analyze the long-run equilibrium numerically. The calibration remains as set out in Table 1. In addition I need to calibrate the fixed entry cost F. To do this, I assume that absent CBDC, the free-entry condition binds. This implies that a value of F can be found from the following equation

$$F = (R_L - R_f)\, L + t_B \frac{1}{N^2}. \tag{40}$$

Figure 4 compares the short-run and long-run responses of deposit rates to changes in the CBDC remuneration rate. The figure shows that the deposit rate is strictly increasing for remuneration rates above but close to $\underline{R}_{CB}$, similar to what we observe in the short-run. This holds for both in the case of no liquidity risk, $\lambda = 0$ and in the case where banks face liquidity risk, $\lambda = 0.1$.

In a long-run equilibrium, as R_{CB} increases, the number of banks in the deposit market decreases. Thus, while banks face additional competition from higher CBDC remuneration rates, this is counteracted in the long-run equilibrium by a more concentrated deposit

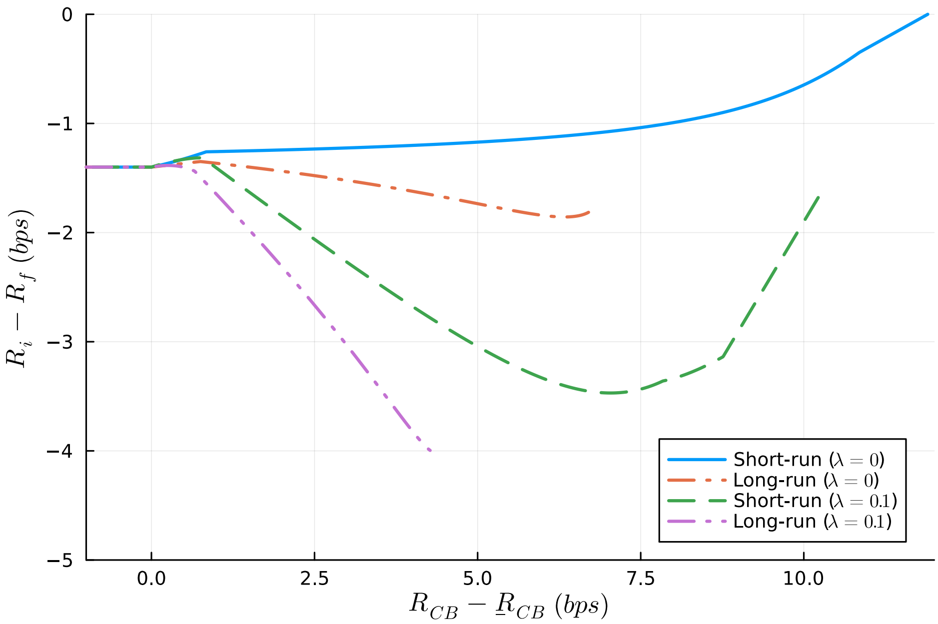

Figure 4: Short-run and long-run impact of R_{CB} on the deposit rate

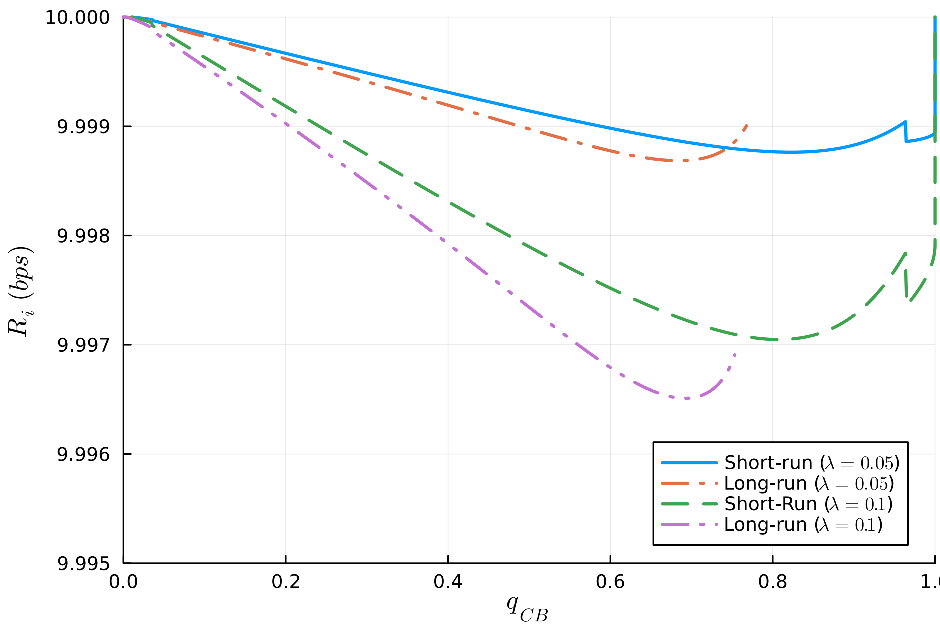

Figure 5: Short-run and long-run impact of CBDC on Monetary Policy Transmission

market resulting in lower competition from other banks. Thus in the long-run there is a dampened response of the deposit rate to changes in the CBDC remuneration rate when compared to the short-run.

The pass-through of the CBDC remuneration rate to the deposit rate is always lower in the long-run than in the short-run equilibrium. Furthermore, in a long-run equilibrium it is possible for the market concentration effect to dominate the increased competition from CBDC, as a consequence even without liquidity risk, $\lambda = 0$, the deposit rate is not guaranteed to be strictly increasing in the CBDC remuneration rate.

Figure 4 further reveals that banks exit the deposit market at a lower remuneration rate in the long-run compared to the short-run. In the short-run, banks exit the deposit market only if, due to high liquidity risk, the marginal benefit of issuing deposits is lower than their marginal cost. In the long-run, banks are subject to a fixed cost of providing deposits and thus banks would exit the deposit market if the benefit of issuing deposits is low enough that they are unable to cover their fixed costs.

I now turn to monetary policy transmission. Absent liquidity risk, $\lambda = 0$, bank profits are unaffected by fully passing on the policy rate change to depositors and thus there is no difference between the short-run and long-run response. Figure 5 compares the short-run and long-run pass-through of a 10bps increase in the policy rate, R_f, to the deposit rate in the case where $\lambda > 0$. As in section 6.3 it is assumed that the CBDC remuneration rate increases in line with the policy rate as specified by equation (35).

The short-run and long-run pass-through is qualitatively similar with two important distinctions. First, Figure 5 shows that when $\lambda > 0$ the pass-through of the policy rate to the deposit rate are further dampened in the long-run compared to the short-run. When banks face liquidity risk, an increase in the policy rate also increases liquidity risk which decreases banks profit and in the long-run results in fewer banks operating in the deposit market. This increased concentration in the banking sector pushes deposit rates lower, resulting in a lower pass-through to deposit rates. Second, Figure 5 also shows that banks exit the deposit market at lower rates of CBDC remuneration and that there is a minimum market share required by banks for entry to be feasible. This minimum market share is increasing in λ as higher liquidity risk lowers the profitability of the banking sector.

7 Conclusion

As the policy debate surrounding the potential introduction of a retail CBDC grows, so does the need for further analysis of its potential implications. This paper focuses on the impact of CBDC on the structure of the market for retail bank deposits and on bank liquidity. In this paper, CBDC is modeled as a source of direct competition for bank deposits. Competition in the deposit market is modeled using a Salop circle model, and thus there is imperfect substitutability between deposits of different banks and the CBDC.

In the absence of liquidity risk, the model suggests that in equilibrium, the introduction of CBDC will result in an increase in interest rates on bank deposits. This leads to a reduction in the market shares of banks in the deposit market. Banks substitute these deposits by obtaining additional liquidity from the central bank through open market operations, and bank profitability falls. Additionally, the model suggests a liquidity risk channel through which CBDC can further increase the costs of banks operating in the deposit market. The liquidity risk faced by banks is increasing in the market share of CBDC and thus this liquidity channel dampens the transmission of the CBDC remuneration rate to the deposit rate. In cases where the liquidity risk is sufficiently large, the bank deposit rate may at some points decrease following an increase in the CBDC remuneration rate. Thus, the paper casts doubt on the use of the CBDC remuneration rate as an additional tool in the central bank monetary policy toolkit.

The paper also highlights the importance of the liquidity risk channel for monetary policy transmission in general. Absent liquidity risk, the model predicts that banks increase the deposit rate one-for-one following an increase in the policy rate, so long as the CBDC remuneration rate is set at a fixed spread from the policy rate. However, if banks face liquidity risk in the deposit market, the introduction of a CBDC also affects the transmission of monetary policy through the bank deposit rate. The impact of a CBDC on the monetary transmission mechanism is greater in the case of an unremunerated CBDC than if the remuneration rate on CBDC was held at a fixed spread with respect to the policy rate.

The model framework allows us to distinguish between the short-run impact of CBDC, where the number of banks is fixed, and the long-run impact where the number of banks may adjust. In the long-run, the model suggests that the introduction of CBDC will reduce the number of banks active in the deposit market and lead to greater concentration in the banking sector. As a consequence, the pass-through of the CBDC remuneration rate to the bank deposit rate is lower in the long-run than in the short-run, and the deposit rate may even be decreasing in the CBDC remuneration rate. Furthermore, the impact of monetary policy impacts the structure of the deposit market, and thus monetary policy will have a different impact in the short- and long-run.

Although this paper makes no claims regarding the welfare implications of the introduction of CBDC, it would be prudent for policymakers to take into account the welfare implications of a more concentrated banking sector that may follow the introduction of a CBDC, as well as possible implications for the transmission of monetary policy.

References

Abad, J., Barrau, G. N., and Thomas, C. (2023). CBDC and the operational framework of monetary policy. BIS Working Papers 1126, Bank for International Settlements.

Abramova, S., Böhme, R., Elsinger, H., Stix, H., and Summer, M. (2022). What can CBDC designers learn from asking potential users? Results from a survey of Austrian residents. Working Paper 241, Oesterreichische Nationalbank.

Agur, I., Ari, A., and Dell'Ariccia, G. (2022). Designing central bank digital currencies. *Journal of Monetary Economics*, 125:62–79.

Andolfatto, D. (2021). Assessing the Impact of Central Bank Digital Currency on Private Banks. *The Economic Journal*, 131(634):525–540.

Bacchetta, P. and Perazzi, E. (2021). CBDC as Imperfect Substitute for Bank Deposits: A Macroeconomic Perspective. Swiss Finance Institute Research Paper Series 21-81, Swiss Finance Institute.

Barrdear, J. and Kumhof, M. (2022). The macroeconomics of central bank digital currencies. *Journal of Economic Dynamics and Control*, 142(104148).

Bindseil, U. (2019). Central bank digital currency: Financial system implications and control. *International Journal of Political Economy*, 48(4):303–335.

Bordo, M. D. (2021). Central bank digital currency in historical perspective: Another crossroad in monetary history. Working Paper 29171, National Bureau of Economic Research.

Böser, F. and Gersbach, H. (2020). Monetary policy with a central bank digital currency: The short and the long term. CEPR Discussion Papers 15322, C.E.P.R. Discussion Papers.

Bouckaert, J. (2000). Monopolistic competition with a mail order business. *Economics Letters*, 66(3):303–310.

Brunnermeier, M. K. and Niepelt, D. (2019). On the equivalence of private and public money. *Journal of Monetary Economics*, 106:27–41.

Bucher, M., Hauck, A., and Neyer, U. (2020). Interbank market friction-induced holdings of precautionary liquidity: Implications for bank loan supply and monetary policy implementation. *Economic Theory*, 70(1):165–222.

Burlon, L., Montes-Galdón, C., Muñoz, M. A., and Smets, F. (2022). The optimal quantity of CBDC in a bank-based economy. Working Paper Series 2689, European Central Bank.

Chiappori, P.-A., Perez-Castrillo, D., and Verdier, T. (1995). Spatial competition in the banking system: Localization, cross subsidies and the regulation of deposit rates. *European Economic Review*, 39(5):889–918.

Chiu, J. and Davoodalhosseini, S. M. (2023). Central Bank Digital Currency and Banking: Macroeconomic Benefits of a Cash-Like Design. *Management Science*, 69(11):6708–6730.

Chiu, J., Davoodalhosseini, S. M., Jiang, J., and Zhu, Y. (2023). Bank Market Power and Central Bank Digital Currency: Theory and Quantitative Assessment. *Journal of Political Economy*, 131(5):1213–1248.

Fernández-Villaverde, J., Sanches, D., Schilling, L., and Uhlig, H. (2021). Central bank digital currency: Central banking for all? *Review of Economic Dynamics*.

Hauck, A. and Neyer, U. (2014). A model of the Eurosystem's operational framework and the euro overnight interbank market. *European Journal of Political Economy*, 34:S65–S82.

Ho, K. and Ishii, J. (2011). Location and competition in retail banking. *International Journal of Industrial Organization*, 29(5):537–546.

Keister, T. and Sanches, D. (2023). Should Central Banks Issue Digital Currency? *Review of Economic Studies*, 90(1):404–431.

Kumhof, M. and Noone, C. (2021). Central bank digital currencies – Design principles for financial stability. *Economic Analysis and Policy*, 71(C):553–572.

Lammersdorf, N., Linzert, T., and Monnet, C. (2023). CBDC, Monetary Policy Implementation, and The Interbank Market. Working paper, Universität Bern.

Madden, P. and Pezzino, M. (2011). Oligopoly on a Salop Circle with Centre. *The B.E. Journal of Economic Analysis & Policy*, 11(1).

Matutes, C. and Vives, X. (1996). Competition for Deposits, Fragility, and Insurance. *Journal of Financial Intermediation*, 5(2):184–216.

Meaning, J., Dyson, B., Barker, J., and Clayton, E. (2021). Broadening Narrow Money: Monetary Policy with a Central Bank Digital Currency. *International Journal of Central Banking*, 17(2):1–42.

Moon, N., McHugh, S., and Cass, N. (2015). Personal Current Account Investigation: A report for the Competition and Markets Authority by GfK NOP. *GfK NOP*.

Niepelt, D. (2020). Reserves for all? Central bank digital currency, deposits, and their (non)-equivalence. *International Journal of Central Banking*, 16(3):211–238.

Park, K. and Pennacchi, G. (2008). Harming Depositors and Helping Borrowers: The Disparate Impact of Bank Consolidation. *The Review of Financial Studies*, 22(1):1–40.

Piazzesi, M. and Schneider, M. (2022). Credit lines, bank deposits or CBDC? Competition & efficiency in modern payment systems. Working paper, Stanford.

Repullo, R. (2004). Capital requirements, market power, and risk-taking in banking. *Journal of Financial Intermediation*, 13(2):156–182.

Salop, S. C. (1979). Monopolistic Competition with Outside Goods. *The Bell Journal of Economics*, 10(1):141.

Williamson, S. (2022). Central bank digital currency: Welfare and policy implications. *Journal of Political Economy*, 130(11):2829–2861.

Appendix

Proof of Proposition 1

By combining (5) and (7) the single bank's problem can be written as follows

$$\pi_i = \max_{B_i, R_i} \left\{ R_L L - R_f B_i - R_i q_i - (1-\lambda) R_{IB}^0 \epsilon_i^0 - \lambda R_{IB}^+ \epsilon_i^+ - F \right\} \tag{A.41}$$

where ϵ_i^+ and ϵ_i^0 are defined by equations (2) and (3) respectively.

Differentiating with respect to B_i yields the following first-order condition

$$-R_f + (1-\lambda) R_{IB}^0 + \lambda R_{IB}^+ = 0. \tag{A.42}$$

Given that $\epsilon_i^0 < \epsilon_i^+$ it follows from equation (4) and $R_f \in (R_{DF}, R_{LF})$ that $R_{IB}^0 < R_{IB}^+$. Furthermore, since obtaining too much or too little liquidity from the central bank in $t = 2$ is costly, the following inequality constraints must hold, $\epsilon_i^0 \leq 0$ and $\epsilon_i^+ \geq 0$, with one of these inequality constraints holding with equality.

Thus, there are two cases to consider. First, if $\epsilon_i^0 = 0$, then banks will have exactly enough liquidity to ensure that if they are in a neutral liquidity position if they do not receive a net liquidity outflow. In this case, $R_{IB}^+ = R_{LF}$ and $R_{IB}^0 \in [R_{DF}, R_{LF})$. From equation (A.42) it follows that the value of R_{IB}^0 that ensures the first condition holds is

$$R_{IB}^0 = R_{LF} - \left(\frac{1}{1-\lambda} \right) (R_{LF} - R_f), \tag{A.43}$$

and that for $R_{IB}^0 \geq R_{DF}$ it must be the case that

$$\lambda \leq \left(\frac{R_f - R_{DF}}{R_{LF} - R_{DF}} \right). \tag{A.44}$$

Finally, for $\epsilon_i^0 = 0$ it follows from equation (3) that

$$B_i = L - q_i. \tag{A.45}$$

The second case to consider occurs if $\epsilon_i^+ = 0$ where banks have exactly enough liquidity so that they do not require additional liquidity should they suffer a net outflow of liquidity. In this case, $R_{IB}^0 = R_{DF}$ and $R_{IB}^+ \in (R_{DF}, R_{LF})$. From equation (A.42) it follows that the value of R_{IB}^{+} that ensures the first condition holds is

$$R_{IB}^{+} = R_{DF} + \frac{1}{\lambda}\left(R_f - R_{DF}\right), \tag{A.46}$$

and for $R_{IB}^{+} < R_{LF}$ it follows that

$$\lambda > \left(\frac{R_f - R_{DF}}{R_{LF} - R_{DF}}\right). \tag{A.47}$$

Finally, for $\epsilon_i^{+} = 0$ it follows from equation (2) that

$$B_i = L - \left(1 - q_{CB}\xi\right) q_i. \tag{A.48}$$

The proposition follows.

Proof of Proposition 2

First, differentiating the bank's profit function given in equation (A.41) with respect to R_i and combining with equation (A.42) gives the following first-order condition for the bank given by equation (20).

In the case where $q_{CB} = 0$, the demand function that the bank faces is given by equation (9) and thus

$$\frac{\partial q_i}{\partial R_i} = \frac{1}{t_B}, \tag{A.49}$$

while from equation (6) if $q_{CB} = 0$ then

$$\frac{\partial E\left[C_i\right]}{\partial q_i} = -R_f, \tag{A.50}$$

and

$$\frac{\partial E\left[C_i\right]}{\partial q_{CB}} = 0. \tag{A.51}$$

With $q_{CB} = 0$, all banks have an equal market share in equilibrium. Since there is full coverage, $q_i = \frac{1}{N}$. Combining the above with the first-order condition yields the following equation for the deposit rate

$$R_i = R_f - t_B \frac{1}{N}. \tag{A.52}$$

Finally, from equation (11), the distance from bank i where a depositor is indifferent between holding a bank i deposit and the CBDC is $x_i^{*} = \frac{1}{2N}$. The highest possible remuneration rate in CBDC such that all depositors prefer bank deposits to CBDC,

$\underline{R}_{CB}$, can be found by combining equations (14) and (A.52) and substituting $x_i^* = \frac{1}{2N}$ to yield

$$\underline{R}_{CB} = R_f - \frac{3}{2} t_B \frac{1}{N}. \tag{A.53}$$

Proof of Proposition 3

An equilibrium with $R_{CB} > R_f - \frac{3}{2} t_B \frac{1}{N}$ can be summarized by the $\bar{x}_i$ that maximizes bank profit. A necessary condition of this is that the $\bar{x}_i$ is the solution to equation (27).

To consider the properties of Γ, first note that from equation (17), the demand for bank i's profits can be written as

$$q_i = \begin{cases} 2x_i^* & \text{if } \bar{x}_i \geq x_i^* + \frac{1}{t_B}\left(\bar{t} - \underline{t}\right) \\ 2x_i^* - \left(\frac{t_B}{\bar{t}-\underline{t}}\right)\left(x_i^* - \bar{x}_i + \frac{1}{t_B}\left(\bar{t} - \underline{t}\right)\right)^2 & \text{if } 0 \leq \bar{x}_i - \frac{1}{t_B}\left(\bar{t} - \underline{t}\right) < x_i^* < \bar{x}_i \\ 2\bar{x}_i - \frac{1}{t_B}\left(\bar{t} - \underline{t}\right) & \text{if } 0 \leq \bar{x}_i - \frac{1}{t_B}\left(\bar{t} - \underline{t}\right) < \bar{x}_i < x_i^* \\ \left(\frac{t_B}{\bar{t}-\underline{t}}\right)\left(2\bar{x}_i - x_i^*\right) x_i^* & \text{if } \bar{x}_i - \frac{1}{t_B}\left(\bar{t} - \underline{t}\right) < 0 < x_i^* < \bar{x}_i \\ \bar{x}_i^2 \left(\frac{t_B}{\bar{t}-\underline{t}}\right) & \text{if } \bar{x}_i - \frac{1}{t_B}\left(\bar{t} - \underline{t}\right) < 0 \leq \bar{x}_i < x_i^* \\ 0 & \text{if } \bar{x}_i < 0 \end{cases} \tag{A.54}$$

It is clear from equation (A.54) that the relevant interval for $\bar{x}_i$ is the interval $\bar{x}_i \in \left[0, x_i^* + \frac{1}{t_B}\left(\bar{t} - \underline{t}\right)\right]$. At the upper-bound where $\bar{x}_i \to x_i^* + \frac{1}{t_B}\left(\bar{t} - \underline{t}\right)$, CBDC has no market share, $q_{CB} = 0$ and banks obtain market share $q_i = \frac{1}{N}$. Substituting this into equation (27), the following can be shown

$$\lim_{\bar{x}_i \to \left(x_i^* + \frac{1}{t_B}\left(\bar{t} - \underline{t}\right)\right)} \Gamma \leq 0 \tag{A.55}$$

and furthermore, wen note that

$$\lim_{\bar{x}_i \to \left(x_i^* + \frac{1}{t_B}\left(\bar{t} - \underline{t}\right)\right)} \Gamma = 0 \iff R_{CB} = \underline{R}_{CB}. \tag{A.56}$$

Next, consider the derivative of equation (27) with respect to $\bar{x}_i$ which can be written as

$$\begin{aligned} \frac{\partial \Gamma}{\partial \bar{x}} = &-\frac{\partial q_i}{\partial \bar{x}} + \left(R_f - R_i\right) \frac{\partial}{\partial \bar{x}} \left\{ \frac{\partial q_i}{\partial R_i} \right\} - \frac{\partial R_i}{\partial \bar{x}} \frac{\partial q_i}{\partial R_i} \\ &- \lambda \xi R_{IB}^+ \left[\frac{\partial q_{CB}}{\partial \bar{x}} \frac{\partial q_i}{\partial R_i} + q_{CB} \frac{\partial}{\partial \bar{x}} \left\{ \frac{\partial q_i}{\partial R_i} \right\} \right] \\ &- \lambda \xi R_{IB}^+ \left[\frac{\partial q_i}{\partial \bar{x}} \frac{\partial q_{CB}}{\partial R_i} + q_i \frac{\partial}{\partial \bar{x}} \left\{ \frac{\partial q_{CB}}{\partial R_i} \right\} \right]. \end{aligned} \tag{A.57}$$

Using equation (27) to substitute out $(R_f - R_i)$ yields

$$\begin{aligned}\frac{\partial \Gamma}{\partial \bar{x}} &= -\frac{\partial q_i}{\partial \bar{x}} + \frac{\partial}{\partial \bar{x}}\left\{\frac{\partial q_i}{\partial R_i}\right\}\left(\frac{\partial q_i}{\partial R_i}\right)^{-1}(q_i + \Gamma) - \frac{\partial R_i}{\partial \bar{x}}\frac{\partial q_i}{\partial R_i} \\ &\quad - \lambda \xi R_{IB}^{+}\left[\frac{\partial q_{CB}}{\partial \bar{x}}\frac{\partial q_i}{\partial R_i} + q_i \frac{\partial}{\partial \bar{x}}\left\{\frac{\partial q_{CB}}{\partial R_i}\right\}\right] \\ &\quad - \lambda \xi R_{IB}^{+}\left(\frac{\partial q_i}{\partial \bar{x}} - \frac{\partial}{\partial \bar{x}}\left\{\frac{\partial q_i}{\partial R_i}\right\} q_i \left(\frac{\partial q_i}{\partial R_i}\right)^{-1}\right)\frac{\partial q_{CB}}{\partial R_i}\end{aligned} \tag{A.58}$$

From equation (A.58), it can be shown that if $\lambda \xi R_{IB}^{+} \leq \frac{3}{2}(\bar{t} - \underline{t})$ whenever $\Gamma < 0$, we have $\frac{\partial \Gamma}{\partial \bar{x}_i} < 0$ and thus lowering $\bar{x}_i$ will increase Γ. Next, it can be shown that for any $\bar{x}_i > 0$, if $\Gamma = 0$ then at that point, $\frac{\partial \Gamma}{\partial \bar{x}_i} < 0$. Thus it follows that there is at most one interior solution to equation (27).

At the lower-bound where $\bar{x}_i \to 0$, CBDC dominates the market, $q_{CB} = 1$ and we will establish that this corresponds to an upper-bound on the CBDC remuneration rate denoted by $\bar{R}_{CB}$. It should be noted that $\bar{x}_i = 0$ is always a solution to equation (27), however, this solution does not always correspond to the profit maximizing deposit rate set by banks.

It follows from the above that if at the point $\bar{x}_i = 0$, $\frac{\partial \Gamma}{\partial \bar{x}_i} \leq 0$, then $\bar{x}_i = 0$ is the unique solution to equation (27) and banks maximize profit by exiting the deposit market and setting $q_i = 0$. If on the other hand at the point $\bar{x}_i = 0$, $\frac{\partial \Gamma}{\partial \bar{x}_i} > 0$, then there exists some $\bar{x}_i \in \left(0, x_i^* + \frac{1}{t_B}(\bar{t} - \underline{t})\right)$ such that $\Gamma = 0$ and this corresponds to the banks profit maximizing.

To find the upper-bound on R_{CB}, note that this will correspond to the point where $\bar{x}_i = 0$ is the unique solution to equation (27). At this point we note that

$$R_i = R_{CB} - \bar{t}. \tag{A.59}$$

For this to be a unique profit-maximizing solution we require that

$$\lim_{\bar{x}_i \to 0} \frac{\partial \Gamma}{\partial \bar{x}_i} < 0. \tag{A.60}$$

where

$$\lim_{\bar{x}_i \to 0} \frac{\partial \Gamma}{\partial \bar{x}_i} = \left(2(R_f - R_{CB} + \bar{t}) - \lambda \xi R_{IB}^{+}\right)\left(\frac{1}{\bar{t} - \underline{t}}\right). \tag{A.61}$$

and thus

$$\lim_{\bar{x}_i \to 0} \frac{\partial \Gamma}{\partial \bar{x}_i} < 0 \iff R_{CB} > R_f + \bar{t} - \frac{1}{2}\lambda \xi R_{IB}^{+} \equiv \bar{R}_{CB}. \tag{A.62}$$

Proof of Proposition 4

From equation (14), in a symmetric equilibrium with $q_{CB} > 0$ the deposit rate can be written as

$$R_i = R_{CB} + t_B \bar{x}_i - \bar{t}. \tag{A.63}$$

Given that $\lambda \xi R_{IB}^{+} \leq \frac{3}{2}\left(\bar{t} - \underline{t}\right)$, any equilibrium with $q_i > 0$ features $R_{CB} > \underline{R}_{CB}$.

Applying the Implicit Function Theorem to equation (31) yields the following equation for the pass-through of the CBDC rate to the deposit rate when x_i^* is fixed

$$\frac{\partial R_i}{\partial R_{CB}} = 1 - t_B \frac{\partial \Gamma / \partial R_{CB}}{\partial \Gamma / \partial \bar{x}_i}. \tag{A.64}$$

Differentiating equation (27) with respect to R_{CB} yields $\frac{\partial \Gamma}{\partial R_{CB}} = -\frac{\partial q_i}{\partial R_i}$.

Therefore, it follows that given $\lambda \xi R_{IB}^{+} \leq \frac{3}{2}\left(\bar{t} - \underline{t}\right)$, whenever $q_i > 0$, $\frac{\partial R_i}{\partial R_{CB}} < 1$ and the pass-through of the CBDC rate to the deposit rate is imperfect.

Next, consider the limit of the pass-through as R_{CB} approaches $\underline{R}_{CB}$ from above.

$$\lim_{R_{CB} \downarrow \underline{R}_{CB}} \left\{ \frac{\partial R_i}{\partial R_{CB}} \right\} = \frac{\left(2 \frac{1}{t_B} \lambda \xi R_{IB}^{+} + 1\right) 2 x_i^*}{1 + \left(2 \frac{1}{t_B} \lambda \xi R_{IB}^{+} + 1\right) 2 x_i^*} > 0 \tag{A.65}$$

Thus, the pass-through is positive (but less than 1) for any x_i^* for R_{CB} above but sufficiently close to $\underline{R}_{CB}$.

In the special case where $\lambda = 0$, from equation (A.58) note that at an equilibrium where $\Gamma = 0$ we have

$$\frac{\partial \Gamma}{\partial \bar{x}} = -\frac{\partial R_i}{\partial \bar{x}} \frac{\partial q_i}{\partial R_i} - \frac{\partial q_i}{\partial \bar{x}} - \left(R_i - R_f\right) \frac{\partial}{\partial \bar{x}} \left\{ \frac{\partial q_i}{\partial R_i} \right\} < -t_B \frac{\partial q_i}{\partial R_i} \tag{A.66}$$

where the inequality follows as all the terms are negative and $\frac{\partial R_i}{\partial \bar{x}} = t_B$. From equation (A.64) the result follows that $\frac{\partial R_i}{\partial R_{CB}} > 0$.

Proof of Proposition 5

Applying the Implicit Function Theorem to equation (31) yields the following equation for the pass-through of the policy rate to the deposit rate in the case where the number of firms and hence x_i^* is fixed

$$\frac{\partial R_i}{\partial R_f} = 1 - t_B \frac{\partial \Gamma / \partial R_f}{\partial \Gamma / \partial \bar{x}_i}. \tag{A.67}$$

From Proposition 3 as long as $\lambda \xi R_{IB}^+ \leq \frac{3}{2} \left(\bar{t} - \underline{t} \right)$

$$\frac{\partial \Gamma}{\partial \bar{x}_i} < 0 \tag{A.68}$$

and that from differentiating equation (36)

$$\frac{\partial \Gamma}{\partial R_f} = -\lambda \xi \left(\left[\frac{\partial q_i}{\partial R_i} \right]^{-1} \frac{\partial q_{CB}}{\partial R_i} q_i + q_{CB} \right) \tag{A.69}$$

which in the case where $0 < \frac{1}{t_B} \lambda \xi R_{IB}^+$ is non-zero and in general the pass-through is imperfect with $\frac{\partial R_i}{\partial R_f} \neq 1$.